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Welded Continuous Frames and Their Components

Progress Report No. 36

INFLUENCE OF PARTIAL BASE FIXITY ON THE
BUCKLING STRENGTH OF RIGID FRAMES

by

Theodore V. Galambos

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SYNOPSIS

The buckling strength of a pinned-base rigid frame is considerably lower than that of an identical fixed-base frame. In current design practice however pinned column bases are specified for most structures because the construction of fixed bases is often very expensive. It is shown in this report that the rotational restraint offered by common "pinned" column bases is sufficient to increase the buckling strength of a single-story, single-bay, and a two-story, single-bay rigid frame almost to the buckling strength of an identical fixed-base structure.

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1. INTRODUCTION

The critical buckling load of a pinned-base rigid frame is considerably below that of an identical fixed-base frame. This may be shown by any one of the many "exact" and approximate methods which are available for determining the buckling strength of rigid frames. Many of these methods are given in Chapter VI and VII of Ref. 1, in Chapter II of Ref. 2, and in Ref. 3, 4, and 5.*

The single-story, single-bay rectangular rigid frame will be examined as an illustration. For simplicity it will be assumed that the axial force acting on each column is equal and that the frame is symmetric. The two frames which will be compared are shown in Fig. 1. Figure 1a shows the pinned-base frame and Fig. 1b shows the fixed-base frame.

The frame shown in Fig. 1 may buckle in one of two modes:

- (1) It may fail by a sidesway buckling mode.
- (2) It may fail in a symmetric mode if sidesway is prevented at the top of the columns.

Of these two, the sidesway type buckling is the more critical (p. 251 of Ref. 1). Therefore this mode of buckling has been selected for computation here. The solid lines in Fig. 1

* References are listed at the end of this report.

represent the stable, undeflected shape of the frame. The dashed lines show the unstable, sideswayed frame.

The problem of the pinned-base, single-story, single-bay rigid frame is solved in Section 73 of Ref. 1, and the corresponding fixed-base frame is solved in Section 74. The critical buckling load of either frame is

$$P_{cr} = \frac{\pi^2 E_t I_C}{(kL_C)^2} \quad (1)$$

where

P_{cr} = The load causing frame buckling

E_t = The tangent modulus

I_C = Moment of inertia of the columns

L_C = Length of the columns

k = A coefficient indicating the "effective" lengths of the columns

The critical buckling condition of the pinned-base frame is expressed by the following buckling equation⁽¹⁾:

$$\frac{\cot \phi}{\phi} - \frac{\gamma}{6} = 0 \quad (2)$$

The corresponding equation for the fixed-base frame is

$$\phi \sin \phi (\cos \phi) \left(\frac{\cot \phi}{\phi} - \frac{\gamma}{6} \right) = 1 \quad (3)$$

In Eqs. 2 and 3 γ is a non-dimensional ratio defined as

$$\gamma = \frac{I_C L_B}{I_B L_C} \quad (4)$$

where

I_B = Moment of inertia of the beam

L_B = Length of the beam

The term ϕ in Eqs. 2 and 3 is equal to

$$\phi = L_C \sqrt{\frac{P_{cr}}{E_t I_C}} \quad (5)$$

The relationship between the coefficients k in Eq. 1 and ϕ in Eqs. 2 and 3 is established as follows: From Eq. 5,

$$P_{cr} = \frac{\phi^2 E_t I_C}{L_C^2} \quad (6)$$

If this value of P_{cr} is equated to P_{cr} of Eq. 1, the relationship between k and ϕ is

$$k = \frac{\pi}{\phi} \quad (7)$$

The curves relating the effective length factor k and the non-dimensional ratio γ , as computed by Eqs. 2 and 3, are given in Fig. 2 for the two column-base conditions. The coefficient k varies from $k = 1.0$ at $\gamma = 0$ to $k = 2.0$ at $\gamma = \infty$ for the fixed-base frame. The corresponding variation

of k for the pinned-base frame is from $k = 2.0$ to $k = \infty$. It must be noted that k enters into the denominator of Eq. 1 as a square term, and thus the reduction in strength due to a pinned-column base is quite considerable according to this theory. Taking for example the case where $\gamma = 1.0$, $k = 1.16$ for the fixed-base frame and $k = 2.33$ for the pinned-end condition. Substitution of these values of k into Eq. 1 shows that the fixed-base frame may carry 4.07 times as much load as the pinned-base frame. As γ increases, the difference tends to become larger. At $\gamma = 5.0$ the ratio of the two buckling strengths is 5.07.

2. STATEMENT OF THE PROBLEM

The foregoing discussion shows that by fixing the column bases, the buckling strength of a rigid frame can be increased by a factor of four or even five. In European practice use is made of this fact by usually specifying fixed-base structures, sometimes at no small expense to the builder. In American practice on the other hand, pinned-base structures are generally the custom.

It is, however, impossible to attain either a fully pinned-end or a fully fixed-end condition in construction. The column bases are in a stage intermediate between the two extremes. The actual restraint depends on the care exercised in the construction of the bases.

The purpose of this report is to investigate the extent of the influence of partial end restraint on the buckling strength of rigid frames. The relationships between base restraint and frame stability will be established to find out how the critical load varies as the restraint changes from full fixity to a fully pinned-end condition. The problem will be solved for two frames by the slope-deflection method described in Section 60 of Ref. 1. These frames are:

- (1) Single-story, single-bay rigid frame
- (2) Two-story, single-bay rigid frame

Base restraint is simulated in the following analysis by inserting a restraining beam between the two column bases as shown in Fig. 3a. This beam restrains the column ends in the same way as would be done by an actual base consisting of base plates, anchor bolts, the footing and the soil. Another way in which the base restraint can be thought to act is shown in Fig. 3b. Base restraint is represented by a set of springs which deform in proportion to the end rotation. ($M = \beta\theta$, where M is the end moment, θ is the rotation, and β is the spring constant.) In the subsequent derivations the frame in Fig. 3a will be analyzed. The method of approach for the system of springs (Fig. 3b) would be similar.

The moment of inertia of the restraining beam in Fig. 3a is I_s . A non-dimensional parameter connecting the properties of this restraining beam to the other dimensions of the

frame is

$$\lambda = \frac{I_S L_C}{I_C L_B} \quad (8)$$

In case the column bases are pinned, no restraint is offered to the columns. Therefore $I_S = 0$ and $\lambda = 0$. For a fixed-base $I_S = \infty$ and $\lambda = \infty$.

3. METHOD OF SOLUTION

The mode of solving the following problems is described in Section 60 of Ref. 1. The method is the slope-deflection procedure. The moments at the ends of a member AB (Fig. 4) are the following:

$$\begin{aligned} M_A &= \frac{E_t I}{L} \left[C\theta_A + S\theta_B - \rho(C + S) \right] \\ M_B &= \frac{E_t I}{L} \left[S\theta_A + C\theta_B - \rho(C + S) \right] \end{aligned} \quad (9)$$

where

θ_A, θ_B = End rotations of member AB

ρ = Rotation of the bar between A and B

It is assumed here that the member has a uniform stiffness along its whole length. The coefficients C and S are defined as

$$C = \frac{c}{c^2 - s^2} \quad (10)$$

$$S = \frac{s}{c^2 - s^2}$$

where

$$\begin{aligned} c &= \frac{1}{\phi^2} (1 - \phi \cot \phi) \\ s &= \frac{1}{\phi^2} \left(\frac{1}{\sin \phi} - 1 \right) \end{aligned} \quad (11)$$

Values of C and S corresponding to practical variations of ϕ are listed in Section 10. This table also shows various other coefficients which will be defined later.

The sign convention of Eq. 9 is the following: Joint rotations θ and bar rotations ϕ are positive when the rotation takes place in a clockwise sense. For example, all joint and bar rotations of the frame in Fig. 3a are clockwise and therefore positive.

A critical buckling equation is obtained by writing an expression for the moment at the ends of each member, using Eq. 9. Equilibrium is maintained at each joint by setting the sum of the moments at the joint equal to zero. Additional necessary equations are obtained from statical conditions. For the frame of Fig. 3a for example, an additional equation is obtained by summing the horizontal reactions and equating their sum to zero. As many independent equations are necessary as the number of unknown joint rotations θ and bar rotations ϕ . The stability condition is then that

the determinant of the coefficients of the unknowns be equal to zero.

The buckling strength of a single-story, single-bay rigid frame will be determined in the next section.

4. THE BUCKLING STRENGTH OF A SINGLE-STORY, SINGLE-BAY FRAME

The frame which is to be analyzed is shown in Fig. 3a. Since the loading and the dimensions are symmetric, only one-half of the frame need be considered. This is because by symmetry $\theta_C = \theta_B$, $\theta_D = \theta_A$ and $\rho_{CD} = \rho_{AB}$. Unknown joint rotations and bar rotations then are θ_A , θ_B , and $\rho_{AB} = \rho$. For these three unknowns, three equations must be found. Joint and bar rotations are clockwise, therefore positive. The moments at the ends of bar AB are, according to Eq. 9, the following:

$$\begin{aligned} M_{AB} &= \frac{E_t I_C}{L_C} \left[C_{AB} \theta_A + S_{AB} \theta_B - \rho (C_{AB} + S_{AB}) \right] \\ M_{BA} &= \frac{E_t I_C}{L_C} \left[S_{AB} \theta_A + C_{AB} \theta_B - \rho (C_{AB} + S_{AB}) \right] \end{aligned} \quad (12)$$

The direction of the moments in Eq. 12 is clockwise as shown in Fig. 4. The bending moments of the end B of bar BC and the end A of bar AC are, (noting that $\rho_{AD} = \rho_{BC} = 0$)

$$\begin{aligned}
 M_{BC} &= \frac{E_t I_B}{L_B} \left[C_{BC} \theta_B + S_{BC} \theta_B \right] \\
 M_{AD} &= \frac{E_t I_S}{L_B} \left[C_{AD} \theta_A + S_{AD} \theta_A \right]
 \end{aligned}
 \tag{13}$$

There is no axial force present in bars AD and BC. Therefore the values of the coefficients C and S are (Eq. 10) equal to

$$\begin{aligned}
 C_{AD} &= C_{BC} = 2 \\
 S_{AD} &= S_{BC} = 4
 \end{aligned}
 \tag{14}$$

If these values of C and S are substituted from Eq. 14 into Eq. 13, and if the subscripts of C and S are dropped in Eq. 12, the equations for the four bending moments are

$$\begin{aligned}
 M_{AB} &= \frac{E_t I_C}{L_C} \left[c \theta_A + s \theta_B - \frac{1}{2}(C + S) \right] \\
 M_{BA} &= \frac{E_t I_C}{L_C} \left[s \theta_A + c \theta_B - \frac{1}{2}(C + S) \right] \\
 M_{BC} &= \frac{6 E_t I_B \theta_B}{L_B} \\
 M_{AD} &= \frac{6 E_t I_S \theta_A}{L_B}
 \end{aligned}
 \tag{15}$$

Next, the equations of equilibrium will be written at the two joints.

(1) Joint A:

$$M_{AB} + M_{AD} = 0 \quad (16)$$

Substituting M_{AB} and M_{AD} from Eq. 15

$$\frac{E_t I_C}{L_C} [c\theta_A + s\theta_B - \rho(c + s)] + \frac{6E_t I_S}{L_B} \theta_A = 0$$

In general E_t of the column will not equal E_t in the beam. For the sake of simplicity, therefore, only elastic buckling will be considered. Then $E_t = E$ in both members, and E may be cancelled out. The equilibrium equation can be rearranged as follows:

$$c\theta_A + s\theta_B - \rho(c + s) + \frac{6I_S L_C}{I_C L_B} = 0 \quad (17)$$

Since $\frac{I_S L_C}{I_C L_B} = \lambda$, Eq. 17 can be written as

$$\theta_A [c + 6\lambda] + \theta_B [s] - \rho[c + s] = 0 \quad (18)$$

(2) Joint B:

$$M_{BA} + M_{BC} = 0 \quad (19)$$

Performing the same operation as for joint A above, the

equilibrium equation will be

$$\theta_A [S] + \theta_B \left[C + \frac{6}{\delta} \right] - \delta [C + S] = 0 \quad (20)$$

Equations 18 and 19 are two equations in the three unknowns θ_A , θ_B , and δ . One additional expression is necessary to obtain three simultaneous equations. This additional equation is obtained from the statical condition that the sum of the horizontal reactions at the bases must equal zero. Since the end moments of column AB are equal in magnitude and sense to the end moments of column CD, the horizontal base reaction must act to the right for each column. This can only be possible if the horizontal reaction of each column is zero.

The additional equation is obtained by summing moments about point B.

$$M_{BA} + M_{AB} + Ph\delta = 0 \quad (21)$$

If the values of M_{BA} and M_{AB} from Eq. 15 are substituted into Eq. 21, the following equation results:

$$\theta_A [C + S] + \theta_B [C + S] - 2\delta(C + S) + \frac{PL_C^2 \delta}{EI_C} = 0 \quad (22)$$

Since $\frac{PL_C^2}{EI_C} = \phi^2$ (Eq. 5), the value of δ is

$$\delta = \left[\frac{C + S}{2(C + S) - \phi^2} \right] (\theta_A + \theta_B) \quad (23)$$

Substitution of ϕ from Eq. 23 into Eq. 18 and 20 reduces the unknowns to θ_A and θ_B . With the abbreviation

$$K = \frac{(C + S)^2}{2(C + S) - \phi^2} \quad (24)$$

these equations reduce to the following two expressions:

$$\begin{aligned} \theta_A [C + 6\lambda - K] + \theta_B [S - K] &= 0 \\ \theta_A [S - K] + \theta_B [C + \frac{6}{\delta} - K] &= 0 \end{aligned} \quad (25)$$

The stability condition is that the determinant of the coefficients must be zero.

$$\begin{vmatrix} (C + 6\lambda - K) & (S - K) \\ (S - K) & (C + \frac{6}{\delta} - K) \end{vmatrix} = 0 \quad (26)$$

The equation for the buckling strength of a rectangular single-story, single-bay frame is then

$$(Q + 6\lambda)(Q + \frac{6}{\delta}) - R^2 = 0 \quad (27)$$

where

$$\begin{aligned} Q &= C - K \\ R &= S - K \end{aligned} \quad (28)$$

It can be shown that Eq. 27 reduces to the buckling condition for the pinned-base frame (Eq. 2) when $\lambda = 0$ and

to the buckling condition for the fixed-base frame (Eq. 3) when $\lambda = \infty$.

The relationship between the effective length factor k and the base fixity is shown in Fig. 5 for several constant values of the ratio λ . The left end of the curves in Fig. 5 corresponds to the values of k for a pinned-end frame, and the right end of the curves approaches the k -values for a fixed-end frame as an asymptote. The curves show that the effective length is rapidly reduced from its value for a pinned-base frame as soon as a small amount of restraint is present. For example, if the base restraint is equal to the stiffness of the top beam ($I_S = I_B$ or $\lambda = 1.0$ if $L_C = L_B$) the values of k are almost equal to their values for a fixed-base frame. This indicates that although the base is still relatively flexible, the small restraint offered by it almost produces the effect of a fully rigid column base.

The effect of partial base fixity on the critical load is shown in Fig. 6. Here the base restraint coefficient λ is plotted versus the critical load P_{cr} (P_{cr} is non-dimensionalized by multiplying it by $\frac{L_C^2}{\pi^2 EI_C}$). It can again be observed that a small amount of restraint at the column ends can increase the critical load considerably.

5. THE BUCKLING STRENGTH OF
A TWO-STORY, SINGLE-BAY FRAME

In order to show that a similar situation to that described above exists for other frames, the buckling strength of one further structure is computed. This is the two-story, single-bay, rectangular rigid frame shown in Fig. 7. The span of this frame is L_C , and the story-heights are the same for each story (story-height = L_C). The moment of inertia is equal to I_C for the columns and I_B for the beams. Base restraint is simulated by a beam which has a moment of inertia I_S .

The buckling strength of the frame shown in Fig. 7 is expressed by the following equation:

$$(Q + 6\lambda)(2Q + \frac{6}{\gamma}) - R^2 (2Q + 6\lambda + \frac{6}{\gamma}) = 0 \quad (29)$$

The derivation of this equation is given in the Appendix. Equation 29 reduces to

$$\frac{6}{\gamma} = \frac{R^2 - Q^2}{Q} \quad (30)$$

for a pinned-base frame ($\lambda = 0$) and to

$$(2Q + \frac{6}{\gamma})(Q + \frac{6}{\gamma}) - R^2 = 0 \quad (31)$$

for a fixed-base frame ($\lambda = \infty$).

The plots of γ versus k for the pinned-base case and the fixed-base case are shown in Fig. 8, and the relationship between k and λ for several constant values of γ is shown in Fig. 9. A rapid decrease of k (and thus an increase of P_{cr}) for a small amount of base fixity can again be observed. The curve for $\gamma = 1.0$ for the single-story bay is shown in Fig. 9.

6. APPLICATIONS OF THE THEORY

In the preceding discussion equations were developed for the critical buckling strength of two symmetrically loaded symmetrical rigid frames. It was shown that a small amount of base restraint, as simulated by an elastic beam between the column bases, reduces the effective length of the columns and thus increases the strength of the frame. The curves in Figs. 5, 6 and 9 indicate that if the ratio

$\frac{I_S}{I_B} \times \frac{L_C}{I_C}$ is greater than one, the buckling strength of the frame is nearly that of a fixed-base frame.

The problem which remains to be discussed is the determination of the restraint offered by actual column bases. This must be known in order to evaluate the stiffness I_S/L_B of the base beam. The relationship between the column base restraint and the stiffness of the fictitious base beam

can be obtained in the following manner: The moment at the end of the base beam is (from Eq. 15),

$$M = \frac{6EI_S\theta}{L_B}$$

If this equation is solved for I_S/L_B , the following relationship exists:

$$\left(\frac{I_S}{L_B}\right) = \left(\frac{M}{\theta}\right) \left(\frac{1}{6E}\right) \quad (32)$$

For convenience it is assumed that the fictitious beam is a steel member for which $E = 30 \times 10^6$ psi. In this case, the stiffness is equal to

$$\left(\frac{I_S}{L_B}\right) = \left(\frac{M}{\theta}\right) \left(\frac{1}{180 \times 10^6}\right) \quad (33)$$

In Eq. 33 the value of M/θ is unknown. This ratio is the initial slope of a curve showing the relationship between a moment and the corresponding rotation for a column base.

Column bases for so-called pinned-base structures consist in general of plates welded to the bottom of the columns. These plates are bolted by anchor bolts to the concrete of the footing. Rotation may take place between the column base and the footing, and the whole footing may rotate in the surrounding soil.

Before any conclusions can be drawn from the theory, it is necessary to know whether this ordinary "pinned"-base arrangement offers sufficient restraint to base rotation to decrease the effective column length towards that of a fixed-base frame.

There are few experimental and analytical studies available which show the moment-rotation characteristics of the footing in the surrounding soil. Roscoe and Schofield⁽⁷⁾ made measurements of the rotation of 8 column footings embedded in sandy soil. (This research was part of the full scale collapse tests of rigid north-light portal frames at Cambridge University.) The footings were designed to furnish full fixity to the column bases, and the experimental moment-rotation curves show that the initial slope of this curve was infinite for most of the columns. No conclusive theory can be deduced from these few tests.

Analytical approximations have been developed for the case where the footing does not rotate in the soil⁽⁶⁾. Rotation takes place between the column ends and the footing due to the deformation of the base plate, the anchor bolts and the concrete. Results of this investigation show that the slope of the moment rotation curve is

$$\frac{M}{\theta} = \frac{bd^2 E_C}{12} \quad (34)$$

where b = width of the base plate

d = length of the base plate

E_C = modulus of elasticity of the concrete

If Eq. 34 is substituted into Eq. 33, the stiffness of the fictitious restraining beam is

$$\frac{I_S}{L_B} = \frac{bd^2}{72n} \quad (35)$$

where n is the modular ratio $\frac{E_{\text{steel}}}{E_{\text{concrete}}}$. The corresponding value of λ is

$$\lambda = \left(\frac{bd}{72n}\right) \left(\frac{L_C}{I_C}\right) \quad (36)$$

In a given design situation all the components of Eq. 36 are known, and λ can be determined. An estimate may be made for the rotation of the footing in the soil, and the value of λ obtained by Eq. 36 can be reduced according to this estimate. The critical buckling load of the frame can be calculated by Eq. 1, where k is obtained from Figs. 5 or 9 for the chosen value of λ .

To show the extent by which partial base fixity influences the buckling strength of a particular frame, an example will be calculated.

A symmetrically loaded, symmetrical, rectangular, single-story, single-bay structure has pinned column bases. Elastic

stress analysis of the frame with the known dimensions L_C and L_B furnishes the following data:

Column size: 8WF31

$$I_C = 170.9 \text{ in}^4$$

$$\gamma = \left(\frac{I_C}{L_C}\right) \left(\frac{L_B}{I_B}\right) = 1.00$$

$$L_C = 156 \text{ in.}$$

Base plate size: 12 in. x 13 in.

If the modular ratio of the foundation concrete is 10, the stiffness of the restraining beam is (Eq. 35)

$$\frac{I_S}{L_B} = \frac{bd^2}{72n} = \frac{(12)^3}{(72)(10)} = 2.40$$

It is now estimated that the footing will rotate in the soil an equal amount to the rotation between the column-ends and the footing. Thus the value of $\frac{I_S}{L_B}$ above will be divided by 2. The corresponding value of λ is then

$$\lambda = \left(\frac{I_S}{L_B}\right) \left(\frac{L_C}{I_C}\right) = \left(\frac{2.40}{2}\right) \left(\frac{156}{170.9}\right) = 1.10$$

The value of k from Fig. 5 is (for $\gamma = 1.00$ and $\lambda = 1.10$): $k = 1.30$. The effective length is then $kL_C = 1.30 \times 156 = 203 \text{ in.}$ The value of k for a pinned-end frame is 2.33, and for a fixed-end frame $k = 1.15$. The critical load of this frame is increased 3.2 times over its critical load if fully pinned-bases are assumed.

7. CONCLUSIONS

The following conclusions may be drawn about the effects of partial base restraint from this analysis:

- (1) Partial base fixity has a beneficial effect on the buckling strength of rigid frames.
- (2) The curves in Figs. 5 and 9 show that for single-bay, single-and two-story rectangular rigid frames a small amount of base restraint reduces the effective length considerably from that of the pinned-base condition.
- (3) Further research on the moment-rotation characteristics of common column foundations is necessary before a more reliable estimate of the base restraint can be made. Available information indicates that presently used "pinned" column bases give enough restraint to increase the buckling strength of the frame by a factor of two or three over the buckling strength computed on the assumption of purely pinned-bases.
- (4) The results of this investigation indicate that the restraint offered by "pinned" column bases is in many cases adequate to prevent failure by frame instability. This fact is especially important for plastically designed structures,

since the buckling strength computed on the basis of pinned-ends often limits the full realization of the plastic collapse mechanism.

The method outlined herein permits the calculation of the critical buckling load of any pinned-base frame, as long as an estimate of the base restraint can be made. For plastic design the method of analysis for determining an approximate buckling load is the following: The plastic hinges of the incomplete mechanism are replaced by real hinges, and a stability analysis is made of this structure. If the critical loads of the structure for which all but the last hinge has formed is higher than or equal to the plastic collapse load, the frame will not fail by frame instability.

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William J. Eney is Director of Fritz Engineering Laboratory and Head of the Department of Civil Engineering.

9. TABLE OF BUCKLING CONSTANTS

ϕ	k	S	C	K	Q	R
0		2.000	4.000	3.000	1.000	-1.000
0.1	31.416	2.000	3.999	3.002	0.997	-1.002
0.2	15.708	2.001	3.995	3.008	0.987	-1.007
0.3	10.472	2.003	3.988	3.018	0.970	-1.015
0.4	7.854	2.005	3.979	3.033	0.946	-1.028
0.6	5.236	2.012	3.952	3.075	0.877	-1.063
0.8	3.927	2.022	3.914	3.137	0.777	-1.115
1.0	3.142	2.034	3.865	3.225	0.640	-1.191
1.2	2.618	2.050	3.804	3.337	0.467	-1.287
1.4	2.244	2.070	3.732	3.491	0.241	-1.421
1.6	1.965	2.093	3.647	3.694	-0.047	-1.601
1.8	1.745	2.120	3.548	3.968	-0.420	-1.848
2.0	1.572	2.152	3.436	4.351	-0.915	-2.199
2.2	1.430	2.189	3.309	4.910	-1.601	-2.721
2.4	1.310	2.233	3.166	5.786	-2.620	-3.553
2.6	1.210	2.283	3.005	7.328	-4.323	-5.045
2.8	1.122	2.343	2.825	10.700	-7.875	-8.457
3.0	1.048	2.412	2.624	23.658	-21.034	-21.246

10. APPENDIXDerivation of the critical equation for a two-story, one-bay frame.

The frame and its dimensions are shown in Fig. 7. Since the frame is symmetric, only one-half the frame need be analyzed. All rotations are clockwise; thus the equations of the moments from Eq. 12 are:

$$M_{AB} = \frac{EI_C}{L_C} \left[C\theta_A + S\theta_B - \rho_1 (C + S) \right]$$

$$M_{BA} = \frac{EI_C}{L_C} \left[S\theta_A + C\theta_B - \rho_1 (C + S) \right]$$

$$M_{BC} = \frac{EI_C}{L_C} \left[C\theta_B + S\theta_C - \rho_2 (C + S) \right]$$

$$M_{CB} = \frac{EI_C}{L_C} \left[C\theta_C + S\theta_B - \rho_2 (C + S) \right]$$

$$M_{CD} = \frac{6EI_B}{L_B} \theta_C$$

$$M_{BE} = \frac{6EI_B}{L_B} \theta_B$$

$$M_{AF} = \frac{6EI_S}{L_B} \theta_A$$

Conditions of Equilibrium:

1. No horizontal shear forces are present.

$$a. \quad M_{BA} + M_{AB} + PL_C \rho_1 = 0$$

$$b. \quad M_{BC} + M_{CB} + PL_C \rho_2 = 0$$

Substitution of the moments yields the following equation for ρ_1 and ρ_2 .

$$\rho_1 = \frac{(C + S)(\theta_A + \theta_B)}{2(C + S) - \phi^2}$$

$$\rho_2 = \frac{(C + S)(\theta_B + \theta_C)}{2(C + S) - \phi^2}$$

2. Equilibrium at joint A:

$$M_{AB} + M_{AF} = 0$$

If the values of the moments and ρ_1 and ρ_2 and the abbreviation K (Eq. 24) are substituted, the following equation results:

$$\theta_A [C - K + 6\lambda] + \theta_B [S - K] = 0 \quad (A-1)$$

3. Equilibrium at joint B:

$$M_{BA} + M_{BC} + M_{BE} = 0$$

Substitution leads to:

$$\theta_A [S-K] + \theta_B \left[2C-2K+\frac{6}{\delta} \right] + \theta_C [S-K] = 0 \quad (A-2)$$

4. Equilibrium at joint C:

$$M_{CB} + M_{CD} = 0$$

Substitution leads to:

$$\theta_B [S-K] + \theta_C \left[C-K+\frac{6}{\delta} \right] = 0 \quad (A-3)$$

Introducing the further simplifications of Q and R (Eq. 28), the three simultaneous equations are:

$$\theta_A \left[Q + 6\lambda \right] + \theta_B [R] = 0$$

$$\theta_A [R] + \theta_B \left[2Q + \frac{6}{\delta} \right] + \theta_C [R] = 0 \quad (A-4)$$

$$\theta_B [R] + \theta_C \left[Q + \frac{6}{\delta} \right] = 0$$

Equating the determinant of the coefficients to zero, the following equation results as the buckling equation:

$$(Q+6\lambda) \left(2Q+\frac{6}{\delta} \right) \left(Q+\frac{6}{\delta} \right) - R^2 \left(2Q+6\lambda+\frac{6}{\delta} \right) = 0 \quad (B-6)$$

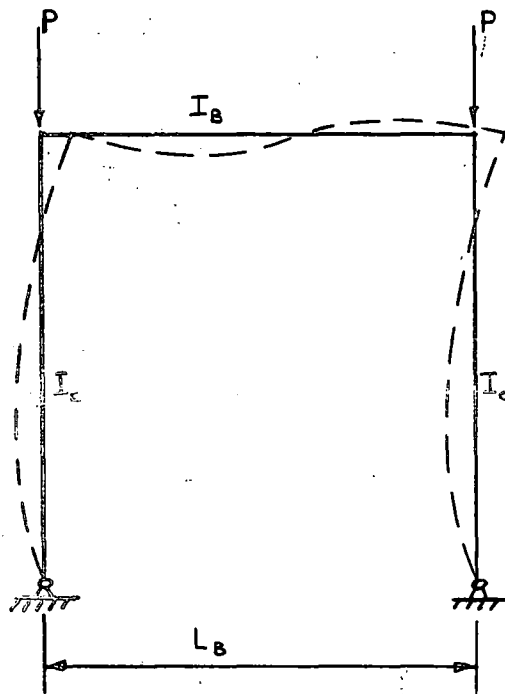


Fig. 1a

PINNED-BASE FRAME

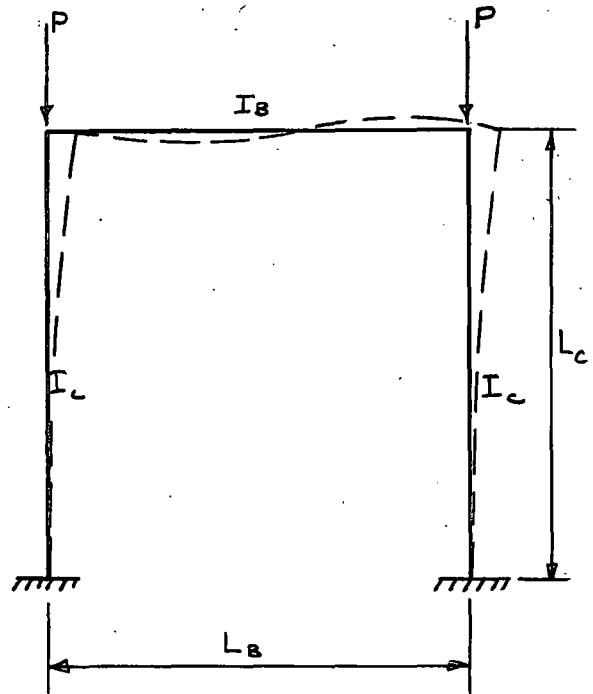
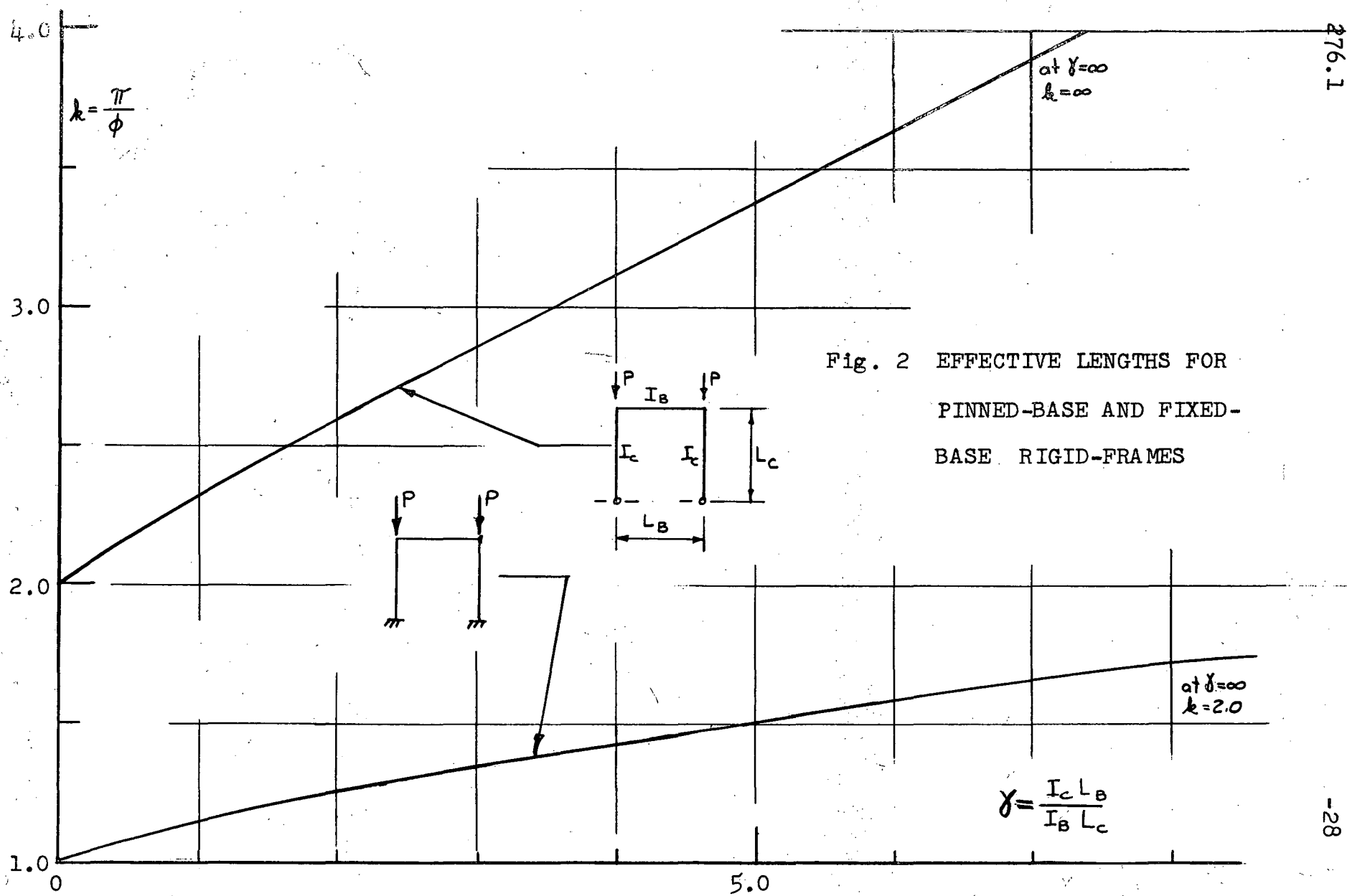


Fig. 1b

FIXED-BASE FRAME

Solid Lines: Stable Deflection Configuration

Dashed Lines: Unstable Deflection Configuration



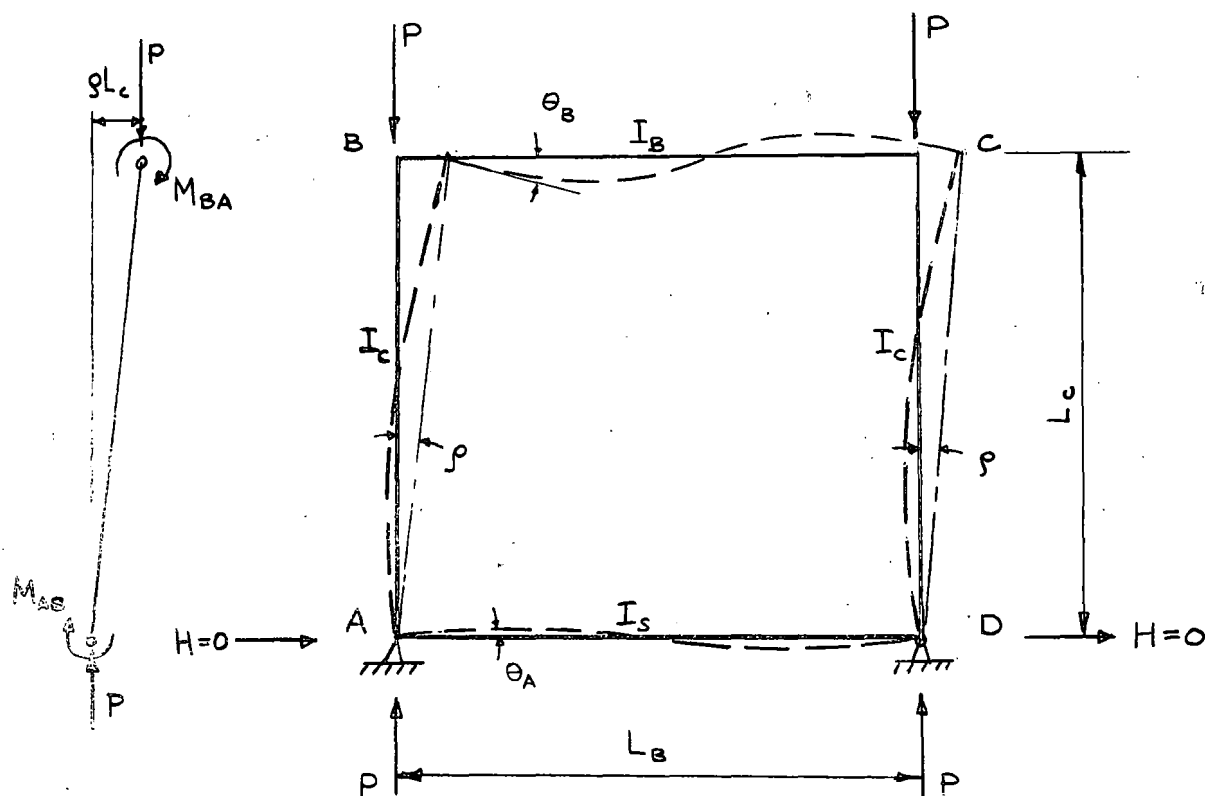


Fig. 3a SINGLE-STORY, SINGLE-BAY, RECTANGULAR
FRAME WITH BASE RESTRAINT

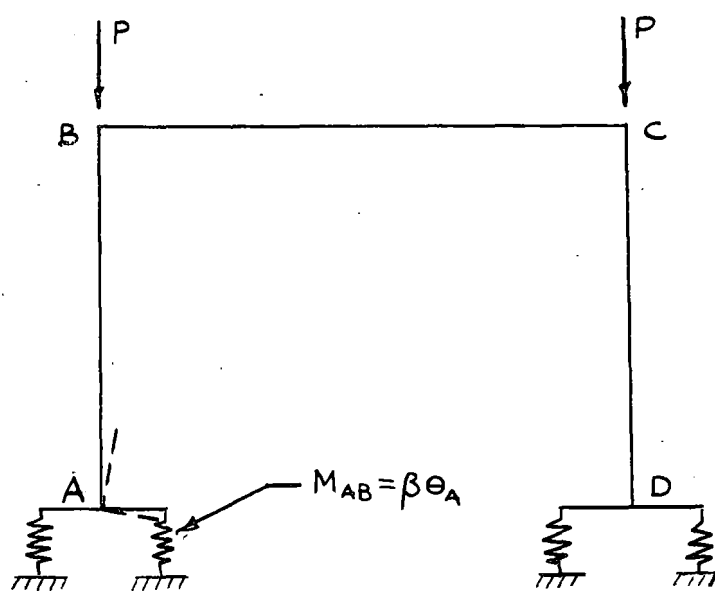
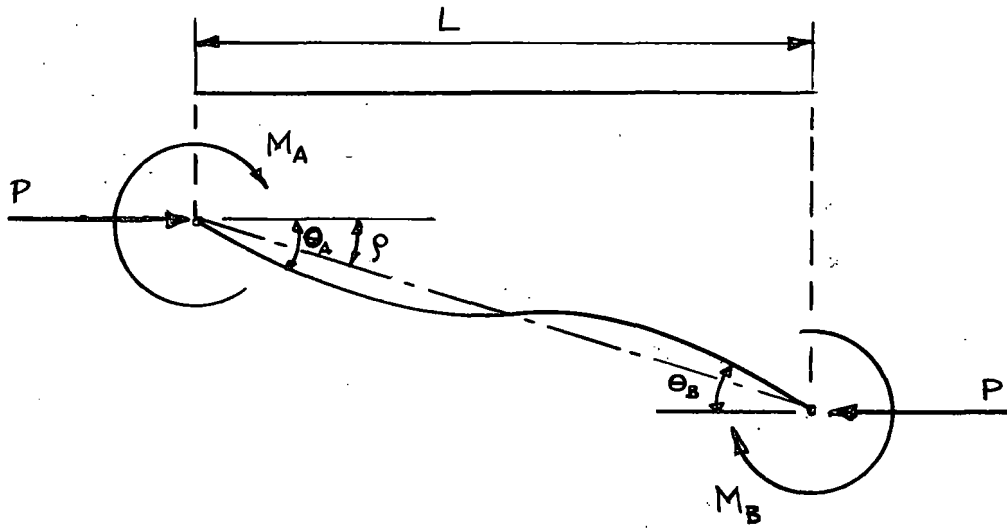


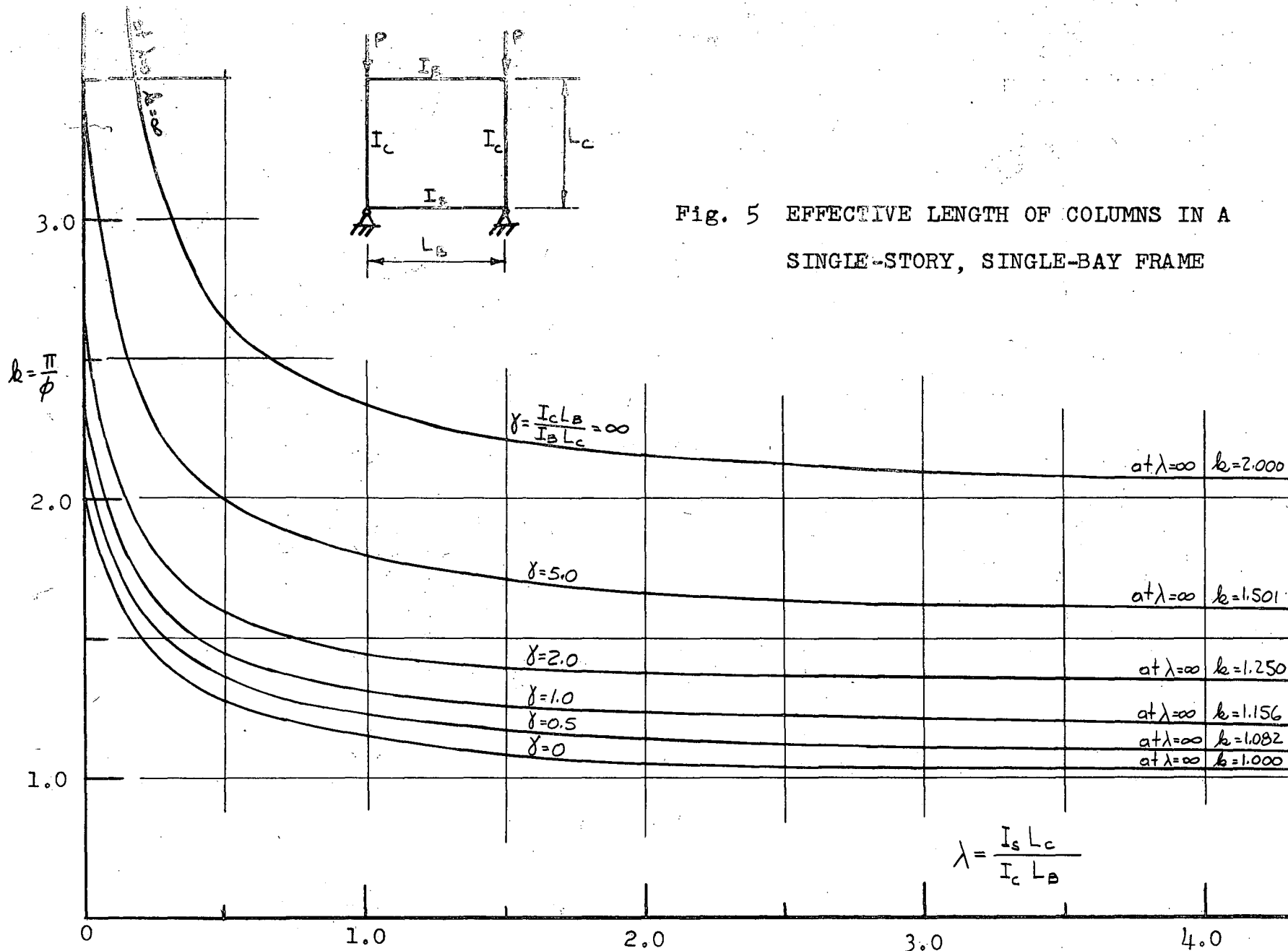
Fig. 3b ANOTHER REPRESENTATION OF BASE FIXITY

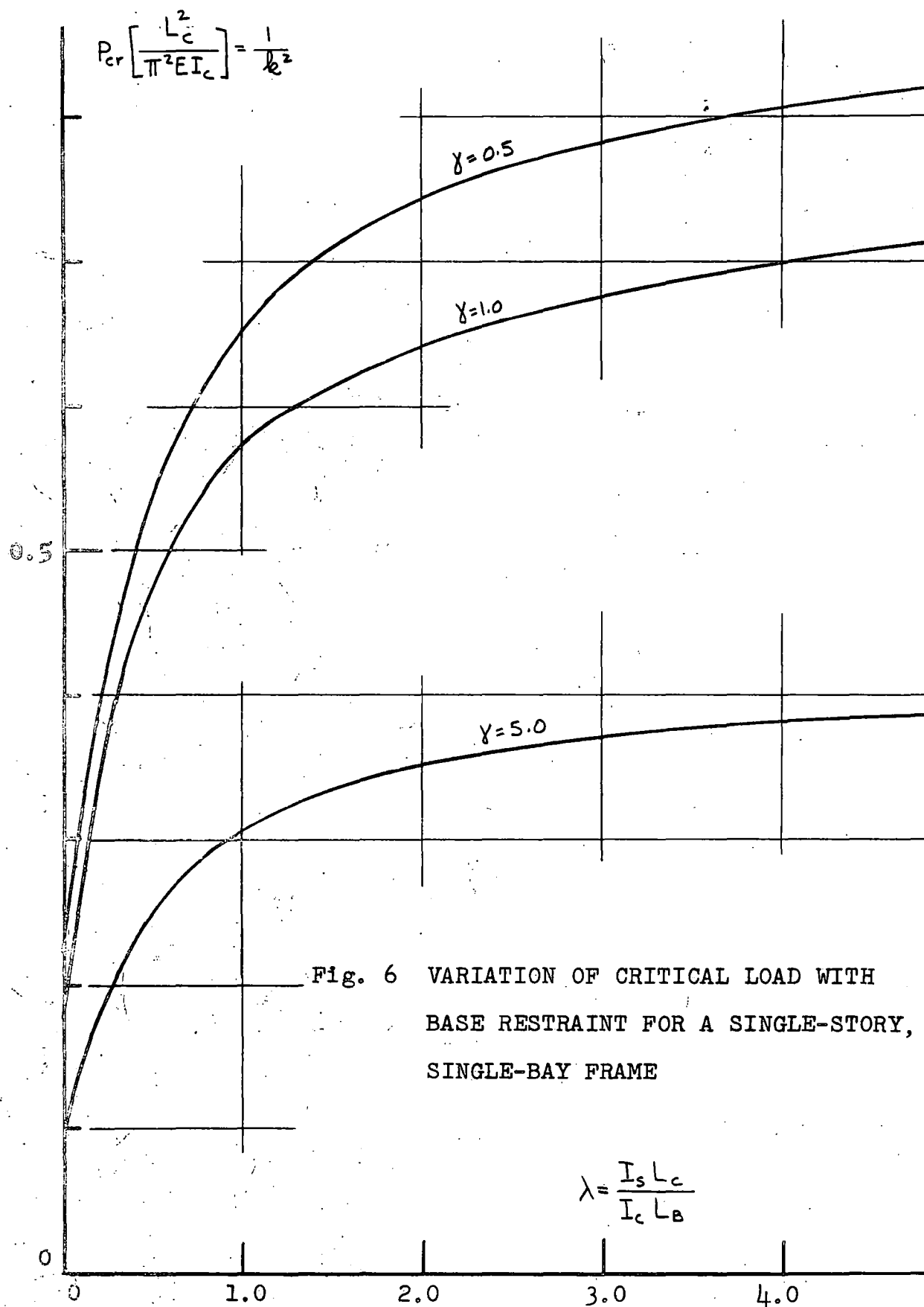


$$M_{AB} = \frac{E_t I}{L} [C\theta_A + S\theta_B - \psi(C+S)]$$

$$M_{BA} = \frac{E_t I}{L} [S\theta_A + C\theta_B - \psi(C+S)]$$

Fig. 4 THE SLOPE-DEFLECTION EQUATIONS





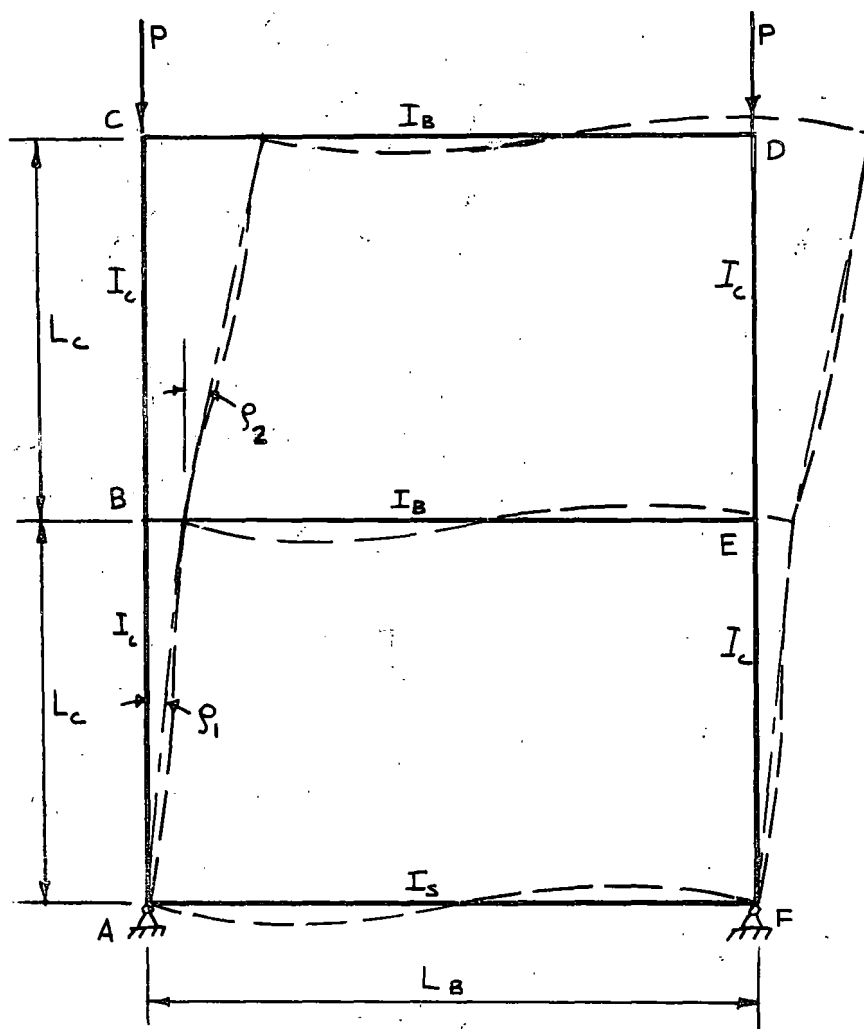


Fig. 7 TWO-STORY, SINGLE-BAY RIGID FRAME

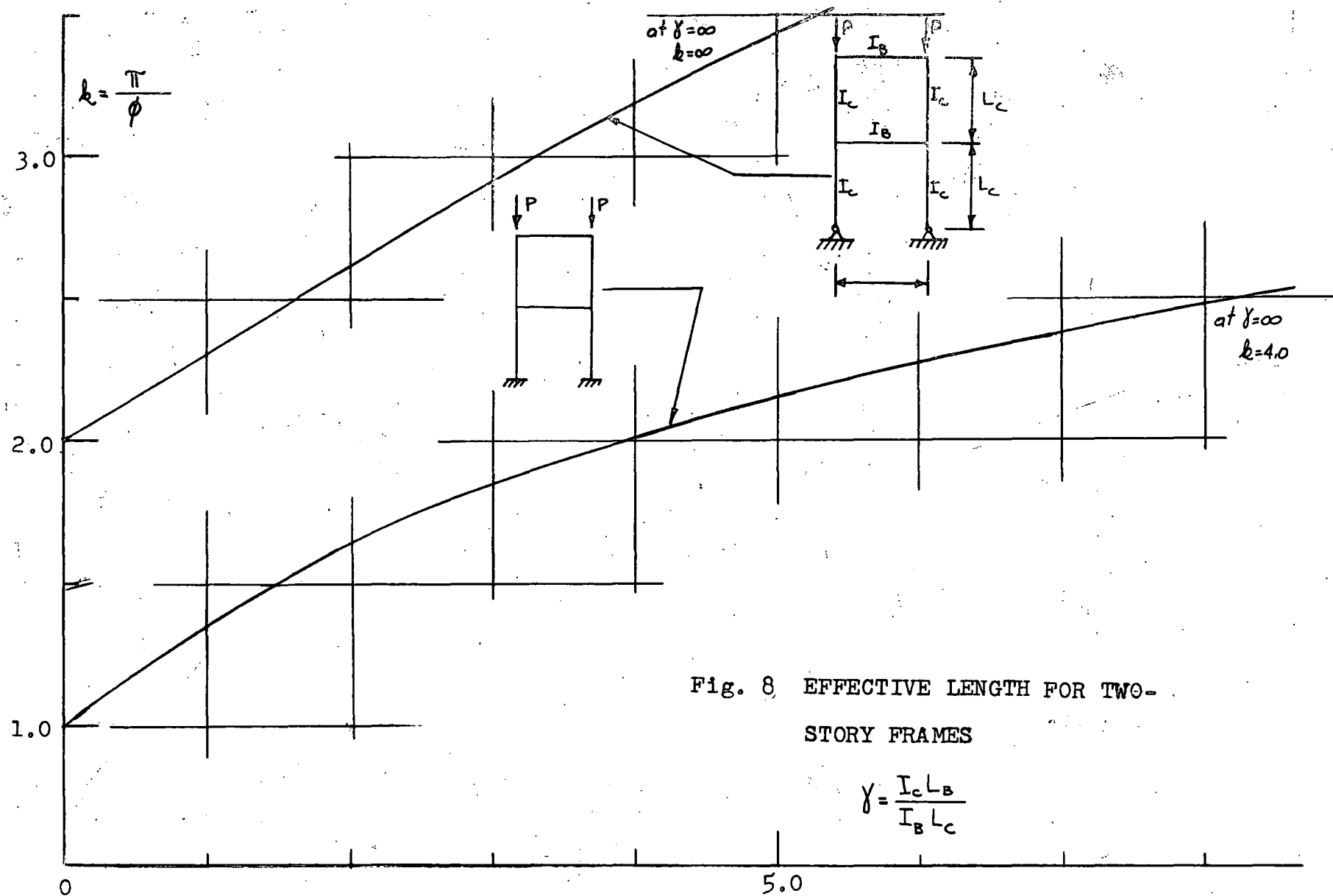
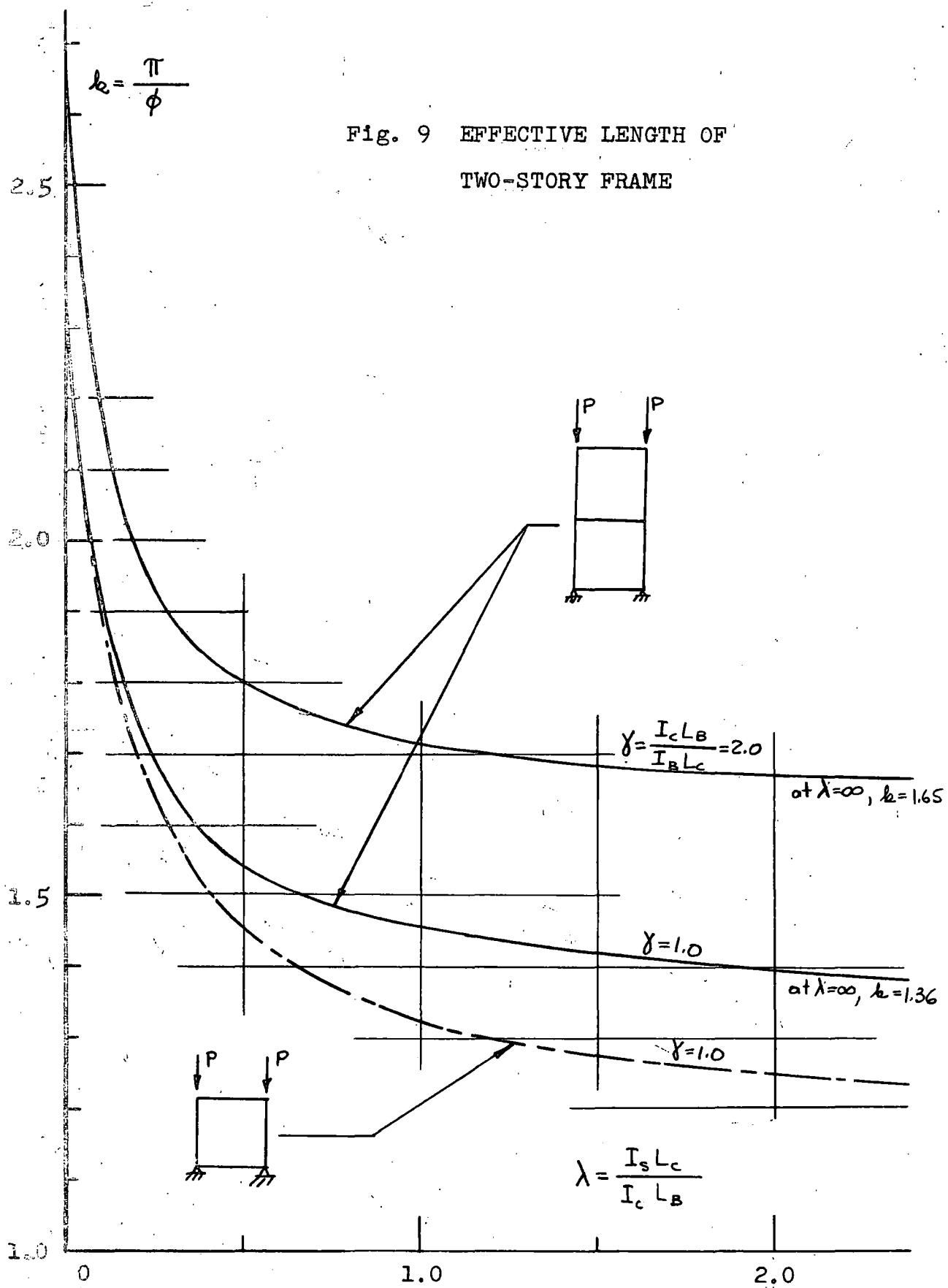
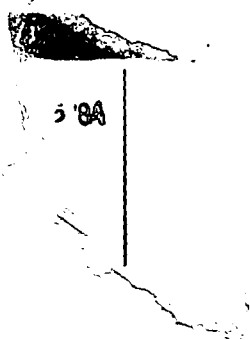


Fig. 8 EFFECTIVE LENGTH FOR TWO-STORY FRAMES



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